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Contribution of Some Particularities in the Dispersion Curves to Numerical Seismograms Computed by Normal Modes*

N. JOBERT

Institut de Physique du Globe, Laboratoire d'Etude Géophysique des Structures Profondes, Université P. et M. Curie, 4Pl. Jussieu, Paris 5e, France

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The first spheroidal higher modes ${}_lS_n$ have stationary values of group velocity for periods larger than 40 sec. Using these modes for model 1066B, theoretical seismograms have been computed in the range of phase velocities of the mantle. A comparison with observation is made for long-period phases recorded after the Peruvian earthquake of October 3, 1974. An agreement is obtained for the arrival of order 4.

INTRODUCTION

The importance of phases related to extrema of group velocity ("stationary phases") of normal modes of the Earth is well-known in long-period seismology. From a theoretical point of view, as Airy phases they spread less than an ordinary group, which results in relative enhancement at large times and distances; moreover, maxima of excitation are related to minima of group velocity [21].

For the fundamental modes, evidence of such phases in the range of period of several minutes, and their persistence and predominance over other phases at large distances, are clear on records of large earthquakes. On the transverse component, the "G" wave is a stationary phase due to a flattening of the group velocity curve; on the vertical and longitudinal components the "R" wave is due to a minimum of group velocity at a period of about 4 min. These features of the dispersion curves are related to physical properties of the upper mantle.

Another example is given by the so-called "S_a" phase, appearing in the range of periods of several tens of seconds for earthquakes of intermediate focal depths; this phase is produced by an interference of several higher modes with a flattening of group velocity in this range of periods [14, 11, 6]. The naming of this phase is due to P. Caloi [5], who thought it a consequence of the presence of an asthenosphere with a low-velocity layer; but it has been shown [23] that this phase is controlled by more general properties of the gradient of velocity S waves in the first hundred km in the upper mantle.

From the point of view of body waves, it could be interpreted by multiple reflections of body waves of the same type at the surface of the Earth, enabling, in presence of

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velocity gradients, interference and coupling between them. The discontinuities of an elastic model (and a free surface particularly) are also a cause of transformation of compressional energy into shear energy and vice versa. Indeed, long-period arrivals, often as strong as those corresponding to the fundamental Rayleigh phases, with group velocities about twice as large, are observed on records of very long period instruments [13]. Long-period body waves have been identified on records of deep earthquakes [1, 17]. We think that an interpretation of interference between long-period multiple body-wave arrivals could be supported by the corresponding existence, and significant contribution, of stationary phases of normal modes.

EFFECT OF THE SURFACE OF THE EARTH AND OTHER DISCONTINUITIES

Models with Plane Layers

Normal modes of free elastic plates have been studied by Tolstoy and Usdin [24]. Their dispersion curves show alternation of maxima and minima of group velocity related to an alternation of compressive and shear modes of vibration. Indeed, at the free surfaces, conversion of P waves into SV waves and the inverse is possible, with variable conditions of reflection.

At the free surface of an elastic stratified half-space, Gilbert [9] has shown how the guided low-frequency PL wave is directly related to the $\bar{P}$ surface wave in Lamb's problem [15] for a homogeneous half-space.

Spherical Models

Extensive studies of spherical layered models have been done by Z. Alterman *et al.* [2, 3]; for spheroidal modes, group velocity extrema appear in the dispersion curves of higher modes. They have been classified into families, with limiting values of velocity at high frequencies, each of which can be related to the time of body-wave arrivals: $P_\mu S_\nu$ (with $\mu \to \infty$) for a homogeneous sphere [2] or $P_\mu(ScS)_\nu$ for a model with a liquid core [3]. On theoretical seismograms these arrivals are clearly apparent and even stronger than the fundamental Rayleigh mode.

These studies show the importance of multiply reflected and converted P waves at the surface of the Earth, and the correspondence between the reflected pulses and higher spheroidal modes.

More realistic models of the Earth take into cccount the variation of physical properties with depth. For the Gutenberg–Bullen A model, Sato *et al.* [22] have shown on theoretical seismograms the existence of a very long-period phase made up of higher spheroidal modes, particularly the third.

The presence of a crust with a low P velocity is required for the building up of PL or $PL(S)$ waves [18]; G. Poupinet [20] has shown that the effect of adding a continental crust to a homogeneous sphere is the appearance of a family of extrema (Fig. 1) on the group-velocity dispersion curves of the spherical model. For this model, corresponding to the two layer continental flat model of Gilbert [9], the position of the extrema are

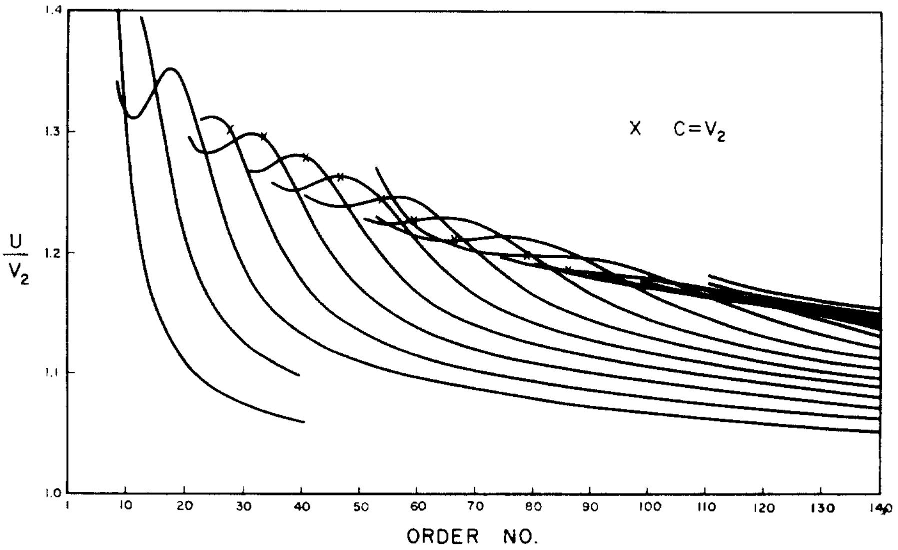

FIG. 1. Variation of group velocities (reduced to the velocity of S waves in the lower medium) with angular order number for a spherical model with a continental layer [20].

on the dispersion curve of the *PL* wave for the flat model, in the range of least attenuation; other families correspond to higher leaking modes.

Figure 2 shows, for periods larger than 40 sec, the dispersion of the first higher spheroidal modes, which have been computed for the model 1066B proposed by Gilbert and Dziewonski [10]. The phase velocities, lower than 16 km/sec, are representative of propagation in the mantle. The branches shown are a continuation toward long periods, and higher group velocities, of those showing extrema of group velocities around 4.5 km/sec, which are characteristic of S_a waves. (A minimum for the third higher mode is seen in the figure.) In the range of periods of 40 sec to 300 sec, rather strong extrema of group velocity appear between 5.5 and 7 km/sec. As the corresponding phase velocities are between 8.5 and 10 km/sec, the modes are related to P rays bottoming between depths of about 200 and 500 km, or SV rays bottoming between depths of 1430 and 2000 km. This phenomenon is thought to be similar to that concerning S_a waves, but the velocities are in the range of P velocities in the upper mantle. We propose as an interpretation the coupling of long-period SV waves traveling through the whole mantle, with long-period P waves traveling only in the upper mantle.

Theoretical Seismograms by Normal Modes

Global comparisons of seismograms computed with normal modes in the range of periods larger than 100 sec with observations of late arrivals, for the deep Colombian earthquake in 1970, have been presented by Luh and Dziewonski [17].

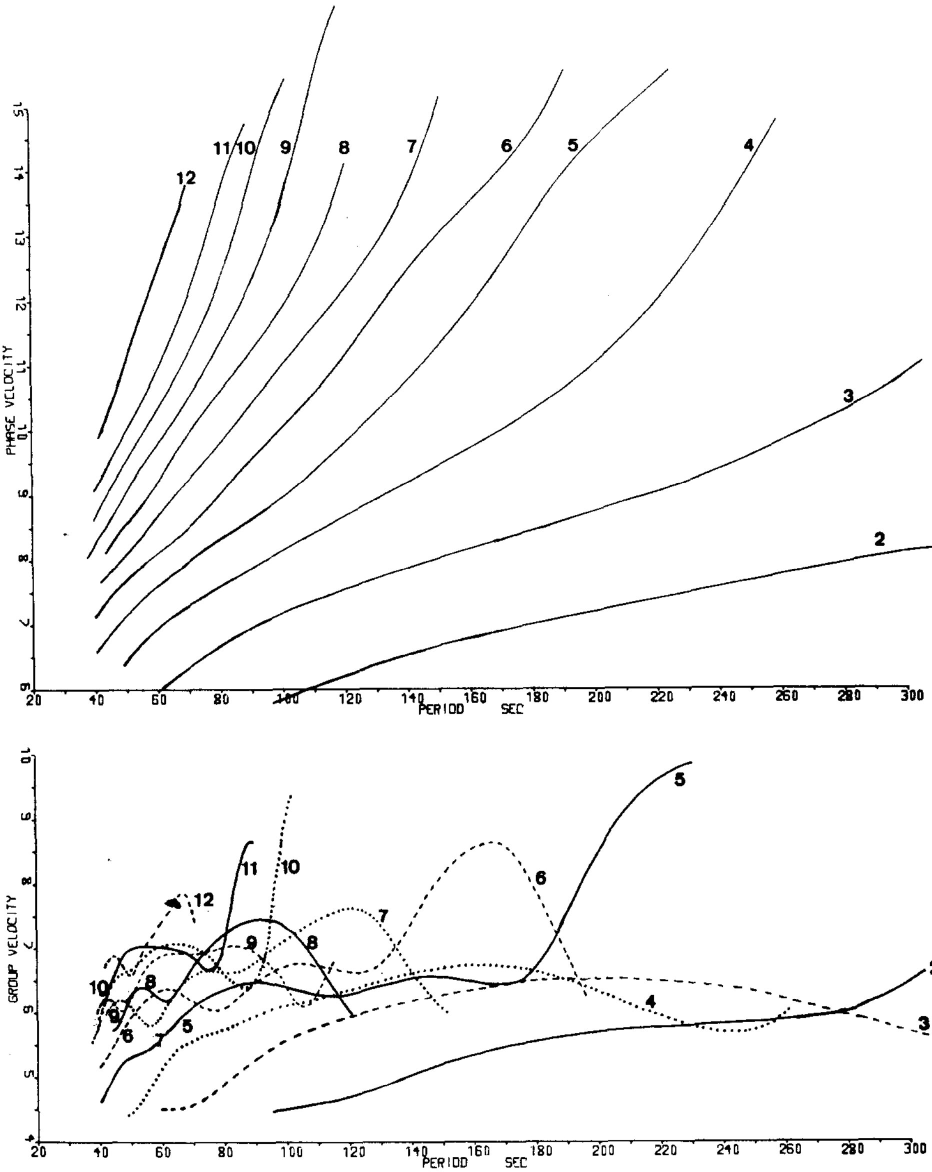

Fig. 2. Phase and group velocity dispersion for model 1066B. The order *l* of the overtones is the usual one at low frequencies; the same branch has been continued at high frequencies, neglecting the Stoneley branch.

Here our main interest is on the first reverse arrivals, which happen to be larger than the corresponding mantle Rayleigh wave arrival (Fig. 5) on the longitudinal component of long-period records of large earthquakes [13].

A theoretical seismogram summing up the modes in the range of periods and velocities shown on Fig. 2 has been computed. As an arrival with an apparent velocity

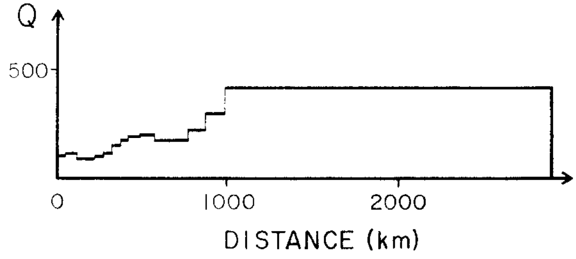

FIG. 3. Variation of Q_μ with depth. The model is from [7] down to 1200 km.

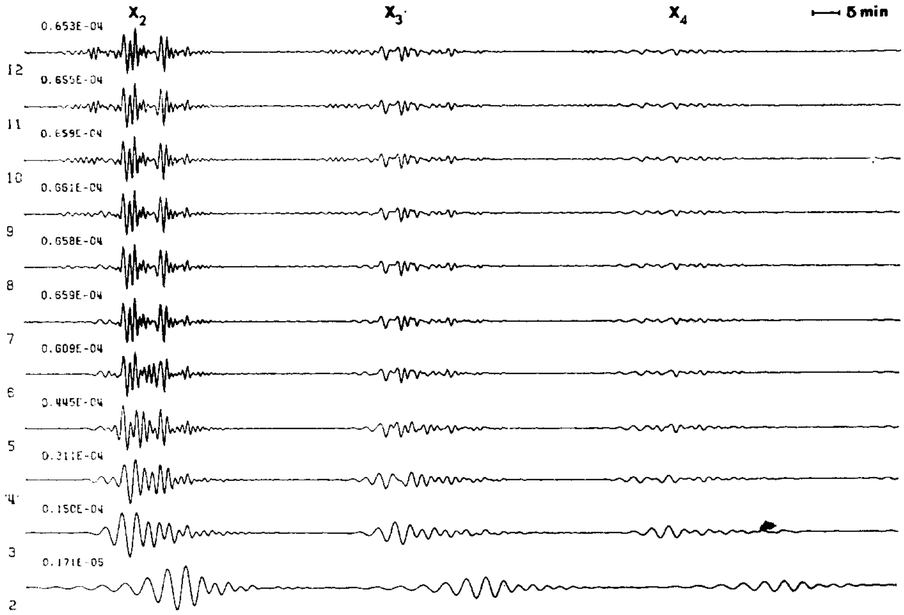

FIG. 4. Theoretical seismogram for the Peruvian earthquake of October 3, 1974: partial sums Σ_2^l of the contribution of radial overtones (instrumental response for the colatitudinal component). Epicentral distance $\Delta = 10054$ km; origin of time is 3500 s. The numbers on the left are the maximal amplitudes of each sum. The amplitudes in each trace have been normalized to the same height.

can be considered as built up by interference of radial overtones with this phase velocity [4, 19], with a cutoff period of 40 sec, twelve modes have been considered enough to construct arrivals with phase velocities in the range of 8.5 to 10 km/sec.

A shear attenuation has been applied to the spheroidal oscillations; we have taken the dissipation model found by inversion of observed damping of free oscillations by A. Deschamps [7]. An extrapolation of the quality factor at a depth of 1200 km has been made in the lower mantle (Fig. 3).

A double couple point source, and Heaviside function of time has been used, with the expressions of [21]. The parameters for this schematic focal mechanism (a thrust

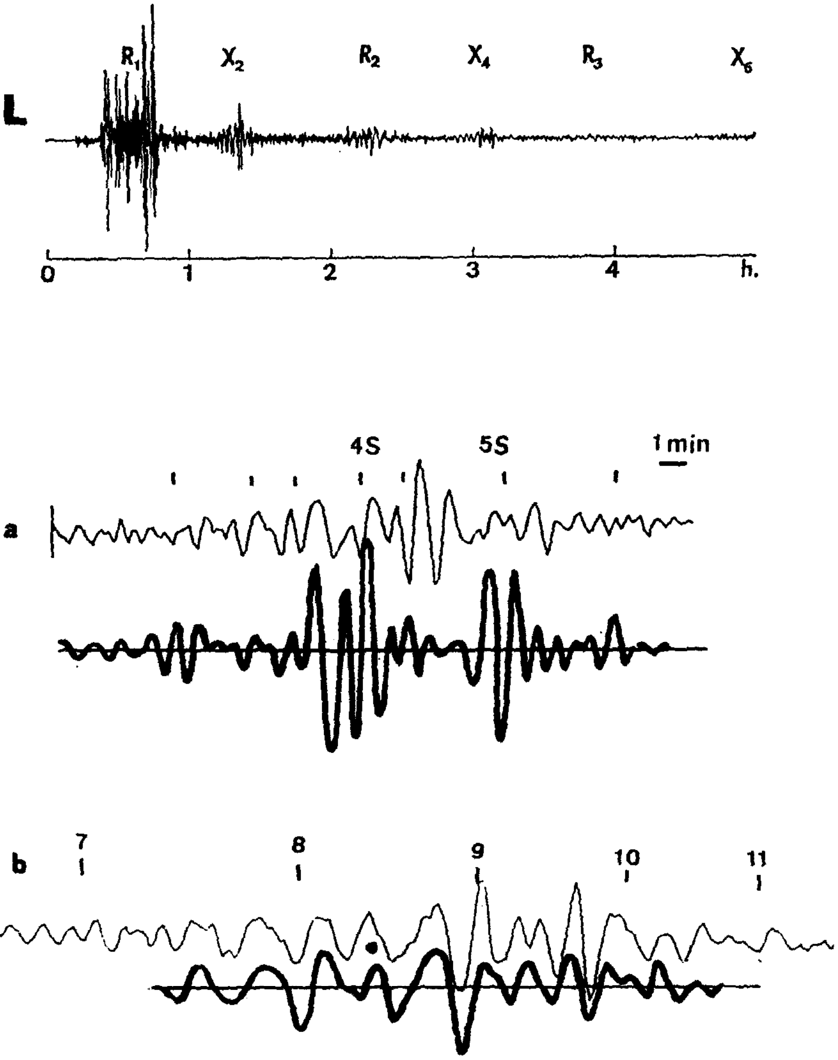

FIG. 5. A reduced copy of the longitudinal-component seismogram for the 1974 Peruvian earthquake is shown at the top. (a) Comparison of the theoretical reverse arrival with the observation (top). The time indications following the arrival time of *nS* are for *n*(*SmP*). (b) The same for arrival of order 4. The dot indicates a velocity of 6.5 km/sec.

fault solution) have been taken from [8] after body-wave observations of the Peruvian earthquake of October 3, 1974. They give, for the fundamental spheroidal mode, theoretical spectra for which the variation of amplitude with colatitudinal order is in rough agreement with the observed spectra at the station of Moulis ($\Delta = 10054$ km). Up to $n = 65$, there is a predominance of even order peaks on the vertical component [12] and of odd order peaks on the longitudinal component. Experimentally, the focal conditions do not seem to be very important for the general appearance of the arrivals under study. But a detailed analysis should take them more precisely into account.

The calculations have been performed for times corresponding to arrivals of the stationary groups of order 2 and more at Moulis. Consistent with the observations, the longitudinal component (Fig. 4) has been found three to four times larger than the vertical component for the sum of overtones. For the ground displacement, the most important contribution is that of the overtones $l = 3$ and 4. Figure 4 shows the partial sums for the instrumental response: The overtone $l = 6$ gives the general shape of the sum, the contributions of higher overtones becoming negligible. In agreement with the observations, the reverse arrival dies off after 5*S* (i.e., *SSSSS*) whereas the weaker vertical component, with spectral energy at shorter periods, presents later multiple *S* phases for the reverse arrival.

Figure 5 shows a comparison of theory with observations for the arrivals of order 2 and 4, well separated from the fundamental mode on the record for the distance considered. For order 2, energy associated with 4*S* and 5*S* is seen, with a predominance of periods of about 60 sec, shorter than those appearing on the observed record. The largest discrepancy is the inability of the computation to show the strongest observed arrivals between these phases. A modification of the global model 1066B used should be tried in order to give a better representation of the crust and upper mantle in the region of propagation of the reverse arrival.

For order 4 (Fig. 5b), the agreement is better; with a slight shift in time, the peaks and troughs are in similar positions.

CONCLUSION

Theoretical seismograms by summation of spheroidal modes have been computed to obtain the contribution of stationary phases with group velocities between 5.5 and 7 km/sec. With the global model used, an agreement with observation could be obtained only for times large enough to allow interference between multiply-reflected long-period body waves. The phases considered, of a type intermediate between body-waves and normal modes, are thought to be influenced by the properties of *P* waves in the upper mantle; a closer comparison between theory and observation of the waveforms of these arrivals could bring information on the physical properties at these depths.

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